Derived Prime-Harmonic Envelope on CA Support

Supplement to the Corrected Robin–MVDC Programme

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June 4, 2026

Abstract

This supplement continues the corrected status PDF manuscript of the Robin–MVDC programme [1]. It records the exact point reached by the reciprocal-prime Route II. First, it recalls the certified B_n^{env}-envelope for $\sigma(n)/n$, explains how it is obtained from the exact divisor product, and then gives a concrete prime-free theta-block construction

$$B_n^{\Theta} = \beta(x) \exp\{-\Phi_{S,r}^{\vartheta}(x)\}$$

for the envelope. This produces the certified Robin reserve used by the block ledger. Starting from the first-moment beta ledger on CA support blocks, the remaining condition is then rewritten as an upper envelope for the prime-harmonic remainder

$$A(x) - B = \sum_{p \le x} \frac{1}{p} - \log\log x - B.$$

The resulting admissible envelope has the form

$$A(x_i) - B \le \frac{C_{\mathrm{req}}^{\Theta}(x_i)}{\sqrt{x_i}\log x_i}$$

on CA support endpoints x_i, where $C_{\mathrm{req}}^{\Theta}(x_i)$ is computed from the theta-block B_n^{Θ}-envelope and the Robin–MVDC ledger. This is a derived requirement, not a proved theorem about the primes. The remaining analytic obstruction is therefore the infinite-range certification of this prime-harmonic oscillation bound on the CA support sequence. Existing explicit reciprocal-prime estimates are reviewed only to clarify why the known all-range bounds are too coarse for this ledger.

Keywords: Riemann Hypothesis; Robin's inequality; colossally abundant numbers; beta envelope; reciprocal primes; prime-harmonic remainder; MVDC; explicit prime estimates.

Status of this supplement. This is a working research note, not a completed proof. It depends on the exact Robin ledger of the corrected status PDF manuscript [1]. It records the exact point reached by the Robin–MVDC Route II reduction: the remaining Robin ledger is reduced to a certified upper envelope for the prime-harmonic fluctuation on CA support endpoints. The envelope is derived by the ledger algebra; the missing theorem is the infinite-range prime-oscillation bound needed to certify that the primes obey it.

1 Relation To The Main PDF

This note is a supplement to the corrected status PDF manuscript [1], specifically to its exact Robin ledger and to Route II, the first-moment reciprocal-prime certificate. The present text does not replace that manuscript. Its role is narrower: it spells out the B_n-envelope used to dominate $\sigma(n)/n$, then follows the Route II algebra until the remaining prime-harmonic oscillation target is made explicit.

The main PDF denotes the actual multiplicative divisor deficit by $A(n)$. In this supplement we write that deficit as $\mathcal{D}(n,x)$, because the letter $A(x)$ is reserved below for the reciprocal-prime remainder

$$A(x) = \sum_{p \le x} \frac{1}{p} - \log\log x.$$

We also reserve B for the Meissel–Mertens prime constant. The divisor-sum upper envelope is therefore written as B_n^{env}.

2 The B_n Upper Envelope

Robin's criterion [2] reduces the Riemann Hypothesis to Robin's inequality above the finite cutoff 5040. The present programme tests the remaining large-candidate ledger on CA/SA-type support profiles, in the spirit of the extremal divisor-sum structure studied by Alaoglu and Erdos [3]. For the CA/SA profiles retained in the corrected ledger, the support is full up to the largest prime $x = P(n)$:

$$n = \prod_{p \le x} p^{a_p}, \qquad a_p \ge 1.$$

For each prime power,

$$\frac{\sigma(p^a)}{p^a} = 1 + \frac{1}{p} + \cdots + \frac{1}{p^a} = \frac{1 - p^{-(a+1)}}{1 - p^{-1}}.$$

Hence

$$\frac{\sigma(n)}{n} = \prod_{p \le x} \frac{1 - p^{-(a_p+1)}}{1 - p^{-1}} = \beta(x) \exp\{-\mathcal{D}(n,x)\},$$

where

$$\beta(x) := \prod_{p \le x} \frac{p}{p-1}, \qquad \mathcal{D}(n,x) := -\sum_{p \le x} \log\left(1 - p^{-(a_p+1)}\right).$$

Let $\mathcal{D}_{\mathrm{env}}(n,x)$ be any certified lower envelope for the actual deficit:

$$\mathcal{D}_{\mathrm{env}}(n,x) \le \mathcal{D}(n,x).$$

Define the divisor-sum envelope

$$B_n^{\mathrm{env}} := \beta(x) \exp\{-\mathcal{D}_{\mathrm{env}}(n,x)\}. \tag{1}$$

This is the B_n-envelope used by the ledger.

Lemma 1 (Certified divisor-sum envelope)**.** *If $\mathcal{D}_{\mathrm{env}}(n,x) \le \mathcal{D}(n,x)$, then*

$$\frac{\sigma(n)}{n} \le B_n^{\mathrm{env}}.$$

Proof. The map $t \mapsto e^{-t}$ is decreasing. Therefore

$$e^{-\mathcal{D}(n,x)} \le e^{-\mathcal{D}_{\mathrm{env}}(n,x)}.$$

Multiplying by $\beta(x) > 0$ and using the exact product formula above gives

$$\frac{\sigma(n)}{n} = \beta(x) e^{-\mathcal{D}(n,x)} \le \beta(x) e^{-\mathcal{D}_{\mathrm{env}}(n,x)} = B_n^{\mathrm{env}}.$$

□

Thus B_n^{env} is not a fitted numerical curve. It is obtained by starting from the exact formula for $\sigma(n)/n$ and replacing the actual divisor deficit by a certified lower bound. The trivial envelope is $\beta(x)$, obtained from $\mathcal{D}_{\mathrm{env}} = 0$. Any stronger certified lower envelope for $\mathcal{D}(n, x)$ lowers B_n^{env} while keeping it above $\sigma(n)/n$.

Robin's inequality follows from the sufficient envelope condition

$$B_n^{\mathrm{env}} \le e^\gamma \log\log n.$$

Taking logarithms and writing

$$E(x) := \log\beta(x) - \gamma - \log\log x, \qquad B_{\log}(n, x) := \log\frac{\log\log n}{\log x},$$

we obtain

$$\log B_n^{\mathrm{env}} - \gamma - \log(\log\log n) = E(x) - \mathcal{D}_{\mathrm{env}}(n, x) - B_{\log}(n, x).$$

Therefore it is sufficient to prove the exact reserve gate

$$E(x) \le \mathcal{D}_{\mathrm{env}}(n, x) + B_{\log}(n, x). \tag{2}$$

The reserve used below is precisely a certified lower bound for the right-hand side of (2) on the CA endpoint under consideration.

2.1 Concrete theta-block construction of B_n^{env}

The preceding argument is abstract: any certified lower envelope $\mathcal{D}_{\mathrm{env}} \le \mathcal{D}$ gives a valid B_n^{env}. We now record the concrete choice used in the present Route II audit. Let $n_{\mathrm{CA}}(x)$ denote a CA support profile with largest support prime $x = p_k$, and let ε be its CA variational parameter. If $q = p_{k+1}$, then the support boundary satisfies

$$\tau_1(q) < \varepsilon \le \tau_1(x), \qquad \tau_s(t) := \frac{\log\Big((1 - t^{-(s+1)})/(1 - t^{-s})\Big)}{\log t}.$$

The Alaoglu–Erdos exponent rule gives

$$a_p \ge s \quad\iff\quad \varepsilon \le \tau_s(p).$$

Moreover

$$\tau_s(p) = \frac{\log\left(1 + \frac{p-1}{p(p^s-1)}\right)}{\log p} < \frac{1}{p^s\log p}.$$

For the large support endpoints used below, Dusart's prime-gap estimate [5] gives a prime in

$$\left(x,\ x + \frac{x}{25\log^2 x}\right].$$

Hence we may put

$$X_+(x) := x\left(1 + \frac{1}{25\log^2 x}\right), \qquad \eta_*(x) := \frac{1}{(X_+(x)+1)\log X_+(x)},$$

and obtain $\varepsilon > \eta_*(x)$. Define layer endpoints $Z_s(x)$ by

$$Z_s(x)^s \log Z_s(x) = \frac{1}{\eta_*(x)}, \qquad Z_1(x) := x. \tag{3}$$

Then for every prime in the layer $Z_s(x) < p \le Z_{s-1}(x)$ we have

$$p^s\log p > \frac{1}{\eta_*(x)} \quad\Longrightarrow\quad \tau_s(p) < \varepsilon \quad\Longrightarrow\quad a_p \le s - 1.$$

Consequently the p-factor in the divisor deficit satisfies

$$-\log\left(1-p^{-(a_p+1)}\right) \geq p^{-s} \qquad (Z_s(x) < p \leq Z_{s-1}(x)).$$

It remains to lower-bound the prime sum in each layer without using the actual prime list. Let

$$\Theta_L(t) \leq \vartheta(t) \leq \Theta_U(t), \qquad \vartheta(t) := \sum_{p\leq t} \log p,$$

be explicit Chebyshev-function bounds. For $1 < A < B$, $s \geq 2$, and a fixed block ratio $r > 1$, set

$$T_0 = A, \qquad T_{j+1} := \min(B, rT_j),$$

and stop at the first J with $T_J = B$. With $[u]_+ := \max(u, 0)$, define

$$\mathcal{T}_{s,r}(A,B) := \sum_{j=0}^{J-1} \frac{[\Theta_L(T_{j+1}) - \Theta_U(T_j)]_+}{(\log T_{j+1})T_{j+1}^s}. \tag{4}$$

Indeed,

$$\vartheta(T_{j+1}) - \vartheta(T_j) \leq \left(\pi(T_{j+1}) - \pi(T_j)\right) \log T_{j+1},$$

so the quantity in brackets gives a certified lower count of primes in the block after division by $\log T_{j+1}$. Every such prime contributes at least T_{j+1}^{-s}. Therefore

$$\sum_{A<p\leq B} \frac{1}{p^s} \geq \mathcal{T}_{s,r}(A,B). \tag{5}$$

Choose a finite order $S \geq 2$. The omitted layers only remove positive terms from a lower bound. Define

$$\Phi^{\vartheta}_{S,r}(x) := \sum_{s=2}^{S} \mathcal{T}_{s,r}(Z_s(x), Z_{s-1}(x)). \tag{6}$$

Combining the exponent-layer bound with (5) gives

$$\Phi^{\vartheta}_{S,r}(x) \leq \mathcal{D}(n_{\mathrm{CA}}(x), x). \tag{7}$$

Thus the concrete certified divisor-sum envelope is

$$B_n^{\Theta} := \beta(x) \exp\{-\Phi^{\vartheta}_{S,r}(x)\}. \tag{8}$$

Proposition 1 (Theta-block B_n-envelope)**.** *For every CA support profile covered by the preceding layer hypotheses,*

$$\frac{\sigma(n_{\mathrm{CA}}(x))}{n_{\mathrm{CA}}(x)} \leq B_n^{\Theta}.$$

Proof. Equation (7) is a certified lower bound for the actual divisor deficit. Substituting $\mathcal{D}_{\mathrm{env}} = \Phi^{\vartheta}_{S,r}$ into Lemma 1 gives the result. □

The corresponding certified reserve in (2) is

$$R_{\Theta}(x) := \Phi^{\vartheta}_{S,r}(x) + B_{\log}(n_{\mathrm{CA}}(x), x). \tag{9}$$

If the exact CA profile is available numerically, one may also form

$$R_{\mathrm{act}}(x) := \mathcal{D}(n_{\mathrm{CA}}(x), x) + B_{\log}(n_{\mathrm{CA}}(x), x).$$

Since $\Phi^{\vartheta}_{S,r}(x) \leq \mathcal{D}(n_{\mathrm{CA}}(x), x)$, we have

$$R_{\Theta}(x) \leq R_{\mathrm{act}}(x).$$

Thus R_Θ is the stricter analytic reserve: it is smaller than the reserve computed from the exact CA profile, but it is obtained without using the prime-by-prime divisor profile as an input.

For the numerical audit below we use the Dusart-type symmetric theta envelope

$$\Theta_L(t) = t(1 - \Delta_\vartheta(t)), \qquad \Theta_U(t) = t(1 + \Delta_\vartheta(t)),$$

where $\Delta_\vartheta(t)$ is the minimum of the following valid terms:

$\Delta_\vartheta(t)$	valid range
1	$t > 1$
$1.2323/\log t$	$t > 2$
$3.965/\log^2 t$	$t > 2$
$0.001/\log t$	$t > 908\,994$
$0.2/\log^2 t$	$t > 3\,594\,641$
$0.05/\log^2 t$	$t > 122\,568\,683$
$0.01/\log^2 t$	$t > 7\,713\,133\,853.$

With $S = 6$ and $r = 1.30$, this gives the concrete B_n^Θ-envelope used in the updated Route II numerical table.

3 Block Objects

Let x_i denote the sampled CA support endpoints and write

$$Y = x_{i-1}, \qquad x = x_i.$$

The reciprocal-prime remainder is

$$A(x) := \sum_{p \le x} \frac{1}{p} - \log\log x,$$

and B denotes the Meissel–Mertens prime constant. The fluctuation to be controlled is therefore

$$A(x) - B.$$

On a CA support block put

$$\nu = \pi(x) - \pi(Y), \qquad H = \log \frac{\log x}{\log Y}, \qquad \mu = \frac{H}{\nu}.$$

Let $R(x)$ be the certified CA reserve at the endpoint x, and let $U(Y)$ be the incoming certified upper ledger at Y. The available block threshold is

$$T(Y, x) := R(x) - U(Y).$$

When the exact CA profile is used only as a numerical diagnostic, we write

$$T_{\text{act}}(Y, x) := R_{\text{act}}(x) - U(Y).$$

When the analytic theta-block B_n^Θ-envelope is used, we write

$$T_\Theta(Y, x) := R_\Theta(x) - U(Y).$$

The incoming ledger $U(Y)$ is the same in both comparisons. Hence

$$T_\Theta(Y, x) = T_{\text{act}}(Y, x) + R_\Theta(x) - R_{\text{act}}(x). \tag{10}$$

4 First-Moment Gate

The beta-error block is

$$Q(Y,x) = \sum_{Y<p\le x} -\log(1-1/p) - \log\frac{\log x}{\log Y}.$$

With

$$u_p = e^{-\mu}\frac{p}{p-1} - 1,$$

we have

$$Q(Y,x) = \sum_{Y<p\le x} \log(1+u_p) \le \sum_{Y<p\le x} u_p =: M_1(Y,x).$$

Writing

$$S_{-1}(Y,x) := \sum_{Y<p\le x} \frac{1}{p-1},$$

the first moment is exactly

$$M_1(Y,x) = e^{-\mu}(\nu + S_{-1}(Y,x)) - \nu. \tag{11}$$

Thus Route II is closed on the block if the sufficient first-moment gate

$$M_1(Y,x) \le T(Y,x) \tag{12}$$

holds. Solving (12) with (11) gives

$$S_{-1}(Y,x) \le \nu(e^{\mu}-1) + e^{\mu}T(Y,x). \tag{13}$$

5 Reduction To $A(x)$

The key exact identity is

$$\frac{1}{p-1} = \frac{1}{p} + \frac{1}{p(p-1)}.$$

Therefore

$$\begin{aligned} S_{-1}(Y,x) &= \sum_{Y<p\le x}\frac{1}{p} + \sum_{Y<p\le x}\frac{1}{p(p-1)} \\ &= \log\frac{\log x}{\log Y} + A(x) - A(Y) + C_2(Y,x), \end{aligned}$$

where

$$C_2(Y,x) := \sum_{Y<p\le x}\frac{1}{p(p-1)}.$$

Substituting this into (13) gives

$$H + A(x) - A(Y) + C_2(Y,x) \le \nu(e^{\mu}-1) + e^{\mu}T(Y,x).$$

Solving for $A(x)$, the first-moment gate is equivalent to

$$A(x) \le A_{\mathrm{req}}(Y,x), \tag{14}$$

where

$$A_{\mathrm{req}}(Y,x) := A(Y) - C_2(Y,x) + \nu(e^{\mu}-1) - H + e^{\mu}T(Y,x). \tag{15}$$

Equivalently, define

$$D(Y,x) := \nu\left(1-(1+\mu)e^{-\mu}\right).$$

Since

$$e^{\mu}D(Y,x) = \nu(e^{\mu}-1) - H,$$

the same required envelope can be written as

$$A_{\mathrm{req}}(Y,x) = A(Y) - C_2(Y,x) + e^{\mu}(T(Y,x) + D(Y,x)). \tag{16}$$

6 The Derived Envelope

Define the normalized required constant by

$$C_{\mathrm{req}}(x) := (A_{\mathrm{req}}(Y, x) - B)\sqrt{x}\log x. \tag{17}$$

This notation is reserve-dependent: A_{req} contains $T(Y, x) = R(x) - U(Y)$. To separate the numerical diagnostic ledger from the analytic theta-block ledger, first set

$$A^{\mathrm{act}}_{\mathrm{req}}(Y, x) := A(Y) - C_2(Y, x) + e^{\mu}(T_{\mathrm{act}}(Y, x) + D(Y, x)), \tag{18}$$

$$A^{\Theta}_{\mathrm{req}}(Y, x) := A(Y) - C_2(Y, x) + e^{\mu}(T_{\Theta}(Y, x) + D(Y, x)). \tag{19}$$

Then define

$$C^{\mathrm{act}}_{\mathrm{req}}(x) := (A^{\mathrm{act}}_{\mathrm{req}}(Y, x) - B)\sqrt{x}\log x, \qquad C^{\Theta}_{\mathrm{req}}(x) := (A^{\Theta}_{\mathrm{req}}(Y, x) - B)\sqrt{x}\log x.$$

By (16), the reserve enters A_{req} only through $e^{\mu}T(Y, x)$. Using (10) therefore gives

$$C^{\Theta}_{\mathrm{req}}(x) = C^{\mathrm{act}}_{\mathrm{req}}(x) + e^{\mu}(R_{\Theta}(x) - R_{\mathrm{act}}(x))\sqrt{x}\log x. \tag{20}$$

Since $R_{\Theta}(x) \le R_{\mathrm{act}}(x)$, the theta-block constant $C^{\Theta}_{\mathrm{req}}(x)$ is smaller. It is the stricter target obtained from the certified B^{Θ}_n-envelope.

Then (14) is exactly the same statement as

$$A(x) - B \le \frac{C_{\mathrm{req}}(x)}{\sqrt{x}\log x}. \tag{21}$$

This is the point reached by the reduction. The right-hand side in (21) is not inserted by hand; it is the admissible fluctuation budget produced by the Robin–MVDC ledger. It says how small the prime-harmonic fluctuation must be on the CA support endpoint x for the Robin ledger to close.

For the analytic B^{Θ}_n-route, the explicit target is therefore

$$A(x) - B \le \frac{C^{\Theta}_{\mathrm{req}}(x)}{\sqrt{x}\log x}. \tag{22}$$

What has not been proved is that the primes satisfy (22) for all sufficiently large CA support endpoints. Thus the remaining obstruction is the prime-core problem:

> certify the upper side of the oscillation of $A(x) - B$ on the infinite CA support range.

7 Why A Fixed C-Scale Matters

The hard content is the scale

$$\frac{1}{\sqrt{x}\log x},$$

not the particular numerical value of a constant. A theorem of the form

$$A(x_i) - B \le \frac{C_*}{\sqrt{x_i}\log x_i}$$

with a fixed constant C_* on all sufficiently large CA support endpoints would be a genuine RH-scale oscillation control. If, in addition,

$$C_* \le C^{\Theta}_{\mathrm{req}}(x_i)$$

eventually, or if the excess $C_* - C^{\Theta}_{\mathrm{req}}(x_i)$ is absorbed by a cumulative ledger margin, then Route II would close.

By contrast, a bound that can be rewritten only as

$$A(x) - B \le \frac{C_{\mathrm{eff}}(x)}{\sqrt{x}\log x}$$

with $C_{\mathrm{eff}}(x) \to \infty$ does not provide the needed oscillation control. It merely expresses a weaker estimate in the same notation.

8 Literature Check

Rosser–Schoenfeld [4] prove the sharp finite-window estimate

$$A(x) \le B + \frac{2}{\sqrt{x}\log x}$$

for $1 < x < 10^8$. This is the origin of the useful $C = 2$ benchmark, but it is not an infinite-range input.

Their all-large reciprocal-prime estimate is of logarithmic scale,

$$|A(x) - B| \le \frac{1}{2\log^2 x},$$

which corresponds in the present normalization to an effective constant

$$C_{\mathrm{eff}}(x) = \frac{\sqrt{x}}{2\log x}.$$

This grows with x, and is therefore too coarse for the Robin–MVDC ledger.

Dusart [6] and Axler [7] give sharper explicit reciprocal-prime estimates in powers of $\log x$. For example Axler's upper bound

$$A(x) - B \le \frac{1}{20\log^3 x} + \frac{3}{16\log^4 x} \qquad (x \ge 46\,909\,074)$$

becomes

$$C_{\mathrm{eff}}(x) = \sqrt{x}\left(\frac{1}{20\log^2 x} + \frac{3}{16\log^3 x}\right)$$

in the $C/(\sqrt{x}\log x)$ scale. This is useful as a finite diagnostic, but it is not a fixed-C RH-scale theorem.

9 Numerical Audit

The support script

```
current_support/robin_mvdc_status_support/test_c2_shortfall_trend.py
```

computes the actual-reserve diagnostic $C^{\mathrm{act}}_{\mathrm{req}}$ and the measured value

$$C_{\mathrm{actual}}(x) := (A(x) - B)\sqrt{x}\log x$$

on sampled CA support endpoints. The updated script

```
test_theta_route_prime_harmonic_threshold.py
```

then substitutes the theta-block B_n^Θ-reserve R_Θ and computes C_{req}^Θ from (20). The following tables are numerical evidence only; they are not a proof of the infinite-range oscillation bound. The corresponding auxiliary scripts and audit data are maintained in the support repository [8].

x	$C_{\text{req}}^{\text{act}}(x)$	$C_{\text{actual}}(x)$	$C_{\text{req}}^{\text{act}}(x) - C_{\text{actual}}(x)$	$C_{\text{req}}^{\text{act}}(x) - 2$
6382007	1.563181	0.770993	0.792188	−0.436819
12253883	1.768796	0.976166	0.792630	−0.231204
29093377	1.629640	0.836043	0.793597	−0.370360
56048351	1.477260	0.697665	0.779596	−0.522740
78580847	1.657538	0.886993	0.770545	−0.342462
108115627	1.789141	0.996545	0.792596	−0.210859
175589599	2.044976	1.253114	0.791862	+0.044976
258144067	1.856973	1.071509	0.785464	−0.143027
339396977	1.438563	0.663823	0.774740	−0.561437
499283831	2.029917	1.272876	0.757041	+0.029917
701361907	1.917137	1.145477	0.771660	−0.082863
966679613	1.699811	0.910342	0.789469	−0.300189

After replacing the exact CA reserve by the theta-block reserve R_Θ, the admissible constant decreases. The stricter audit is

x	$C_{\text{req}}^{\text{act}}(x)$	$C_{\text{req}}^{\Theta}(x)$	$C_{\text{actual}}(x)$	$C_{\text{req}}^{\Theta}(x) - C_{\text{actual}}(x)$
6382007	1.563181	0.850094	0.770993	0.079100
12253883	1.768796	1.089638	0.976166	0.113472
29093377	1.629640	0.980350	0.836043	0.144308
56048351	1.477260	0.862847	0.697665	0.165182
78580847	1.657538	1.045210	0.886993	0.158216
108115627	1.789141	1.189125	0.996545	0.192579
175589599	2.044976	1.464313	1.253114	0.211199
258144067	1.856973	1.284244	1.071509	0.212735
339396977	1.438563	0.871745	0.663823	0.207921
499283831	2.029917	1.472536	1.272876	0.199660
701361907	1.917137	1.369861	1.145477	0.224384
966679613	1.699811	1.161780	0.910342	0.251438

Thus, on the sampled range, the measured fluctuation is still below the stricter theta-block admissible envelope. The smallest observed margin is 0.079100, at $x = 6\,382\,007$. The difficulty is not the finite arithmetic in this table, but the missing proof that the theta-block envelope and the prime-harmonic oscillation control remain strong enough on the infinite CA support range.

10 Final Analytic Target

Analytic Target 1 (CA-support prime-harmonic envelope)**.** *Prove that for all sufficiently large CA support endpoints x_i,*

$$A(x_i) - B \le \frac{C_{\text{req}}^\Theta(x_i)}{\sqrt{x_i}\log x_i},$$

where $C_{\text{req}}^\Theta(x_i)$ is the admissible ledger constant obtained from the certified theta-block B_n^Θ-envelope by (20)*.*

This target is exactly what remains after the Route II reduction with the analytic B_n^Θ-reserve. It is not an MVDC algebra problem anymore; it is a certified oscillation-control problem for the reciprocal-prime remainder on the CA support sequence, coupled to the requirement that the theta-block deficit envelope remains sufficiently tight.

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